4. **Compton effect**

An incident photon with wave vector $\mathbf{k}_0$ is scattered from a particle with rest mass $m_0$ and initial momentum $\mathbf{p}_0 = 0$. The scattered photon has the wave vector $\mathbf{k}$.

Based on the conservation of the relativistic energy and momentum, determine the shift $\lambda - \lambda_0$ of the wave length of the scattered photon as a function of the scattering angle $\theta$ between $\mathbf{k}$ and $\mathbf{k}_0$.

(Remark: Special relativity yields the relation $E^2 = c^2 p^2 + m^2 c^4$ between the energy $E$ and the momentum $\mathbf{p}$ of an object of rest mass $m$.)

5. **Bohr model**

Consider the motion of an electron of charge $-e$ in the electrostatic field of a proton of charge $+e$ according to classical mechanics.

(a) Determine the effective potential for the motion of the electron around the nucleus as a function of the radial distance.

(b) According to the Bohr model, electrons move on circular orbits and the angular momentum $L$ can assume the values $L = n\hbar$, $n \in \{1, 2, \ldots\}$. Determine the possible energies $E_n$, orbital radii $r_n$, and velocity ratios $v_n/c$, where $c$ is the speed of light.

(c) Bohr assumed that only radiation of frequency

$$\omega(n_1, n_2) = \frac{E_{n_1} - E_{n_2}}{\hbar}, \quad n_1 > n_2$$

(1)

can be emitted. Compare $\omega(n + 1, n)$ for $n \to \infty$ with the orbital frequency which results from classical mechanics using the results of part (b).

(d) For velocities $v \ll c$, the power radiated by an accelerated charge is given by the classical expression

$$P = \frac{e^2}{6\pi\varepsilon_0 c^3} \dot{v}^2$$

(2)

in SI units. For the orbit corresponding to energy $E_1$, estimate the time after which the electron (considered to be a classical particle) would hit the nucleus.
6. **Fourier transform**

Given a function $f : \mathbb{R} \to \mathbb{C}$, its Fourier transform $\tilde{f}$, provided it exists, is defined by

$$\tilde{f}(q) := \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} f(x) \exp(-iqx).$$

(3)

The following relations hold:

$$f(x) = \int_{-\infty}^{\infty} \frac{dq}{\sqrt{2\pi}} \tilde{f}(q) \exp(iqx) \quad \text{(inverse Fourier transform)}$$

(4)

$$\int_{-\infty}^{\infty} dx |f(x)|^2 = \int_{-\infty}^{\infty} dq |\tilde{f}(q)|^2 \quad \text{(Parseval’s theorem).}$$

(5)

Consider the Gaussian

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-x^2}{2\sigma^2}\right).$$

(6)

Calculate the Fourier transform $\tilde{f}$, plot $f(x)$ and $\tilde{f}(q)$, and check Eqns. (4) and (5).